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**SUBJECT:** An Analysis of Apollo 8 Tracking Data Utilizing the Osculating Lunar Elements Program - Case 310

**DATE:** June 30, 1969

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TM-69-2014-8

## 1.0   INTRODUCTION

During the Lunar Parking Orbit (L.P.O.) phase of the Apollo 8 mission (after the LOI-2 trajectory circularization), eight front side passes of Doppler tracking data were acquired by the Manned Space Flight Network (M.S.F.N.) and Deep Space Network (D.S.N.).  Post flight analysis made on these data are of particular interest from a navigation viewpoint since the L.P.O. achieved by Apollo 8 closely approximates the lunar orbital conditions in future landing missions.

An analysis of the Apollo 8 MSFN and DSN Doppler tracking data was made using the Osculating Lunar Elements Program (O.L.E.P.).*  In order to evaluate the O.L.E.P. approach to orbit determination, these data were processed on a two pass regression and a two pass propagation zone basis. This memorandum presents an analysis of the results obtained.

## 2.0   MATHEMATICAL BACKGROUND

The basic concept of O.L.E.P. is the representation of some of the low-eccentricity orbital elements, $a$, $e_c = e \cos \omega$, $e_s = e \sin \omega$, $\Omega$, $I$, $m = M+\omega$, by linearly time-varying functions.  During the orbit determination process, estimates are obtained for both the six low-eccentricity elements and some of their corresponding linear time dependent terms.  The actual data reduction is accomplished using a classical least squares algorithm which minimizes the error or residual obtained from processing k observations by differentially correcting the n solution parameters $\{\alpha_i\}$ as follows:

$$\Delta \alpha_i = \left[ \sum_{i=1}^{k} J^T(t_i) W(t_i) J(t_i) \right]^{-1} \sum_{i=1}^{k} J^T(t_i) W(t_i) \Delta \lambda (t_i)$$

*Bullock, M. V. and A. J. Ferrari, "An Analysis of Lunar Orbiter III Tracking Data Utilizing Osculating Orbital Elements," BCM TM-69-2014-2, March 26, 1969, Case 310

where $\Delta\lambda(t_i) = \lambda(t_i) - \hat{\lambda}(t_i)$

In this expression $\Delta\alpha_i$ is the differential correction vector (nx1), $J(t_i)$ is a row vector (1xn) containing the partial derivatives of the observable with respect to the parameters which are to be estimated (evaluated at time $t_i$), $J^T$ is the transpose of the $J$ vector, $W(t_i)$ is the weighting matrix (1x1), $\lambda(t_i)$ is the $t_i^{th}$ observation, and $\hat{\lambda}(t_i)$ is the estimated observable at time $t_i$.

The processing of k observations and the resulting set of differential corrections $\{\Delta\alpha_i\}$ constitute one computing iteration. The convergence criterion for any two successive iterations is as follows:

$$\frac{\sum\limits_{i=1}^{k}\left[\Delta\lambda(t_i)\right]^2 \Big|_{(I-1)}}{\sum\limits_{i=1}^{k}\left[\Delta\lambda(t_i)\right]^2 \Big|_{(I)}} - 1 > \delta$$

where $\delta$ is a small positive number and (I-1) and (I) designate the (I-1) and (I) computing iterations.

The semi-major axis does not appear in O.L.E.P. as an explicit solution parameter. The estimate for the linear portion of the modified anomaly (e.g., $m(t) = m_0 + m_1 t$) is used to imply a corresponding semi-major axis by using the classical Kepler relationship

$$m_1 = \sqrt{\frac{\mu}{a^3}}$$

or

$$a = \left[\frac{\mu}{m_1^2}\right]^{1/3}$$

where $\mu$ is the Newtonian constant times the lunar mass.

3.0  ANALYSIS OF RESULTS

Prior to the analysis of Apollo 8 data, a few explanatory comments should be made.  A comparison of O.L.E.P. two pass regression/two pass propagation solutions was made with those obtained from a standard orbit determination program using R2 lunar gravity field (unpublished results). In almost all cases considered, O.L.E.P. showed noticeably smaller errors (25%) in the two pass propagation zone. Quality assessments made in this memorandum about O.L.E.P. solutions refer to the comparison of propagation zone errors with those of the regression zone.  No attempt is made to compare O.L.E.P. solutions to those obtained from the standard methods.

The eight post-LOI-2 passes of Apollo 8 tracking data are designated 3-10.  (see Table I for tracking periods). Two pass solutions were obtained using O.L.E.P. for the following tracking periods:  Passes: 3-4, 4-5, 5-6, 6-7, 7-8.  Each of these solutions was then propagated into the following two passes of data (e.g., 3-4 → 5-6).

The fundamental parameter set chosen for each two pass regression zone consists of the following eight terms:

$$e_c(t): \quad e_{co}, \ e_{cl}$$

$$e_s(t): \quad e_{so}, \ e_{sl}$$

$$m(t) \ : \quad m_o, \ m_1$$

$$\Omega(t) \ : \ \Omega_o \ (\Omega_1 \text{ fixed at lunar rotation rate})$$

$$I(t) \ : \ I_o$$

This particular parameter set was chosen since it reduced the correlations existing in the covariance matrix between the constant elements and the linear terms to a minimum.  A sample correlation matrix was generated for passes 4-5 with a full 10 parameter set; $e_{co}, \ e_{cl}, \ e_{so}, \ e_{sl}, \ \Omega_o, \ \Omega_1, \ I_o, \ I_1, \ m_o, \ m_1,$ (see Table II).  In eliminating the linear inclination term, $I_1$, and the linear nodal term, $\Omega_1$, two of the more highly correlated terms were deleted from the solution set.  However, several highly correlated terms still

exist in the basic constant element set: $\rho = .997(\Omega_o, I_o)$, $\rho = .997 (I_o, m_o)$, $\rho = .99999(\Omega_o, m_o)$. Since these constant element terms are fundamental to the orbit definition, they were not deleted from the solution set.

In order to facilitate the analysis, an indicator is given with each two pass increment of data processed. It should be noted that these quality assignments are made on a peak-to-peak error basis and are relative in nature. The symbols are presented to give the reader an appreciation for the general quality of the fit during both the regression zone (R.Z.) and the propagation zone (P.Z.). The indicators utilized define the following error bounds:

$$O \leq \max \{E\} \leq 1 \text{ fps} : \text{V.G.}  \text{(Very Good)}$$

$$1 \text{ fps} < \max \{E\} \leq 2 \text{ fps} : \text{G. (Good)}$$

$$2 \text{ fps} < \max \{E\} : \text{P. (Poor)}$$

where max $\{E\}$ is the largest peak-to-peak error. Figures 1-5 are attached showing the error distributions from two typical MSFN/DSN tracking stations for each regression and propagation zone. These figures are presented to display the systematic quality of the errors and most importantly to show the dynamic growth properties in each propagation zone.

|  | R.Z. | P.Z. |
|---|---|---|
| A. Passes: 3,4 R.Z./5,6 P.Z. (Fig. 1) | V.G. | G. |

1. Regression Zone: Solution has a small negative mean of -.0167 fps. Positive and negative error distribution is fairly symmetric.

2. Propagation Zone: Growth in positive errors by a factor of three. The errors in the propagation zone possess a positive mean of .5506 fps. Best extrapolation obtained from Apollo 8 data.

|  | R.Z. | P.Z. |
|---|---|---|
| B.  Passes:  4,5 R.Z./6,7 P.Z. (Fig. 2) | V.G. | P. |

   1.  Regression Zone:  Good solution on an error distribution basis; almost symmetrically distributed errors with a small negative mean of -.0133 fps.

   2.  Propagation Zone:  Solution does not extrapolate well.  Errors grow by an order of magnitude in the positive direction.  The errors in the extrapolated zone have a positive mean of 1.146 fps.

|  | R.Z. | P.Z. |
|---|---|---|
| C.  Passes:  5,6 R.Z./7,8 P.Z. (Fig. 3) | V.G. | P. |

   1.  Regression Zone:  Errors possess an extremely symmetric distribution and a small negative mean of -.0103 fps.

   2.  Propagation Zone:  A very weak solution in forward propagation.  Errors show a growth of 1,2 orders of magnitude in the positive direction.  Errors possess a mean of 1.513 fps.

|  | R.Z. | P.Z. |
|---|---|---|
| D.  Passes:  6,7 R.Z./8,9 P.Z. (Fig. 4) | V.G. | P. |

   1.  Regression Zone:  Solution possesses a small negative mean of -.0153 fps.  Errors have a slightly larger negative distribution.

   2.  Propagation Zone:  Positive growth in errors is about an order of magnitude.  Only small growth in the negative direction.  Errors possess a positive mean of 1.186 fps.

|  | R.Z. | P.Z. |
|---|---|---|
| E.  Passes:  7,8 R.Z./9,10 P.Z. (Fig. 5) | V.G. | P. |

   1.  Regression Zone:  Errors have a slightly negative distribution and possess a small negative mean of -.0178 fps.

   2.  Regression Zone:  Again errors show a large positive growth by about a factor of eight.  Propagation zone errors possess a positive mean of 1.286 fps.

A compilation of the statistical qualities (mean, standard deviation, and root sum of squares) of both the regression and propagation zone errors is given in Table III.

## 4.0 DISCUSSION

The one outstanding characteristic common to all the data intervals processed by O.L.E.P. is the large dynamic growth of errors from the regression to the propagation zone. In order to study this effect the high correlations existing between some of the six constant portions of the modified orbital elements must be considered.

The orbit of Apollo 8 was near circular and possessed a selenographic inclination of about 167°. Since the orbital element set used eliminates by definition the problems connected with near circular orbits, the inclination of Apollo 8 was considered as the potential problem. As previously mentioned, the correlation between the constant part of the longitude of the ascending node, $\Omega_o$, and the modified anomaly, $m_o$, is $\rho = .99999$. This extremely high correlation reveals that the minimization process considers these parameters to be identical and differentially corrects them accordingly. High correlations of this type are a direct result of a lack of sensitivity in the $[J^T W J]^{-1}$ matrix. The most harmful effect of this high correlation is a loss of uniqueness in the minimization process. This loss of uniqueness in the orbital element set becomes amplified when the regression zone solutions are extrapolated. The two other high correlations, $\rho = .9971$ $(I_o, \Omega_o)$, and $\rho = .9972$ $(I_o, m_o)$ also contribute significantly to the loss of solution uniqueness.

## 5.0 CONCLUSIONS

Apollo 8 Doppler tracking data acquired by the MSFN and DSN was analyzed using O.L.E.P. on a two pass regression zone and two pass propagation zone basis. The fundamental parameter set which optimized the O.L.E.P. processing of Apollo 8 data consists of the following parameters: $e_{co}$, $e_{c1}$, $e_{so}$, $e_{s1}$, $\Omega_o$, $I_o$, $m_o$, and $m_1$.

Solutions obtained in the two pass regression zone manifested a characteristic growth in the propagation zone errors by about a factor of five (on a peak to peak basis). This large rate of growth in the errors is due to some high correlations existing between the inclination, ascending node,

and modified anomaly parameters. It is felt that these correlations are attributable to a lack of sensitivity in the solution covariance matrix, $[J^T WJ]^{-1}$. The presence of high correlations indicates that linear combinations existing among the solution parameters lead to non-unique orbital states. This lack of uniqueness in the regression solutions is responsible for the extrapolation characteristics achieved. However, comparing solutions obtained on Apollo 8 data by O.L.E.P. and by standard methods (unpublished results), on the average O.L.E.P. achieves two pass forward propagation with 25% smaller errors. The O.L.E.P. concept provides an effective means of attaining precision navigation in lunar orbit. Work will be continued.

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Attachments
Table I-III
Figures 1-5

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TABLE I

| Pass No. | Apollo 8 Data Included in O.L.E.P. Processing | |
| --- | --- | --- |
| | Starting Time | Ending Time |
| 3 | 24 Dec. 1968 $14^h41^m30^s$ | 24 Dec. 1968 $15^h51^m30^s$ |
| 4 | 24 Dec. 1968 $16^h40^m30^s$ | 24 Dec. 1968 $17^h49^m30^s$ |
| 5 | 24 Dec. 1968 $18^h41^m30^s$ | 24 Dec. 1968 $18^h48^m30^s$ |
| 6 | 24 Dec. 1968 $20^h38^m30^s$ | 24 Dec. 1968 $21^h47^m30^s$ |
| 7 | 24 Dec. 1968 $22^h35^m30^s$ | 24 Dec. 1968 $23^h45^m30^s$ |
| 8 | 25 Dec. 1968 $0^h34^m30^s$ | 25 Dec. 1968 $1^h44^m30^s$ |
| 9 | 25 Dec. 1968 $2^h32^m30^s$ | 25 Dec. 1968 $3^h33^m30^s$ |
| 10 | 25 Dec. 1968 $4^h30^m30^s$ | 25 Dec. 1968 $5^h31^m30^s$ |

TABLE II

Pass 4,5 Correlation

| | $e_{so}$ | $e_{sl}$ | $e_{co}$ | $e_{cl}$ | $I_o$ | $I_1$ | $\Omega_o$ | $\Omega_1$ | $m_o$ | $m_1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $e_{so}$ | 1.000 | -.8026 | .5337 | -.4719 | .6467 | -.5586 | .6581 | -.5769 | .6582 | -.5788 |
| $e_{sl}$ | | 1.000 | -.4144 | .4855 | -.5045 | .5179 | -.5147 | .55381 | -.5147 | .5534 |
| $e_{co}$ | | | 1.000 | -.8590 | .5882 | -.4645 | .6308 | -.5055 | .6300 | -.5046 |
| $e_{cl}$ | | | | 1.000 | -.4833 | .4574 | -.5185 | .4999 | -.5178 | .4989 |
| $I_o$ | | | | | 1.000 | -.8505 | .9971 | -.8565 | .9972 | -.8563 |
| $I_1$ | | | | | | 1.000 | -.8442 | .9962 | -.8444 | .9963 |
| $\Omega_o$ | | | | | | | 1.000 | -.8553 | .9999 | -.8550 |
| $\Omega_1$ | | | | | | | | 1.000 | -.8554 | .9999 |
| $m_o$ | | | | | | | | | 1.000 | -.8551 |
| $m_1$ | | | | | | | | | | 1.000 |

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TABLE III

| Pass Nos. | Mean (fps) | | Standard Deviation (fps) | | Root Sum of Squares (fps) | |
|---|---|---|---|---|---|---|
| | R.Z.* | P.Z.** | R.Z. | P.Z. | R.Z. | P.Z. |
| 3,4 | -.01627 | 0.5506 | .2013 | .4592 | .2020 | .7170 |
| 4,5 | -.01334 | 1.146 | .1924 | 1.297 | .1788 | 1.731 |
| 5,6 | -.01032 | 1.513 | .1958 | 1.567 | .1972 | 2.179 |
| 6,7 | -.01534 | 1.186 | .2036 | 1.122 | .2042 | 1.632 |
| 7,8 | -.01779 | 1.286 | .2068 | 1.098 | .2099 | 1.691 |

 * Regression Zone
** Propagation Zone

BERMUDA MSFN

DOPPLER ERRORS (FPS)

1.2

0.8

0.4

0

4,000   8,000   12,000   16,000   20,000   24,000

TIME (SECONDS)

R.Z. ———————— P.Z. ————————

ASCENSION MSFN

DOPPLER ERRORS (FPS)

1.2

0.8

0.4

0

-0.4

4,000   8,000   12,000   16,000   20,000   24,000

TIME (SECONDS)

R.Z. ———————— P.Z. ————————

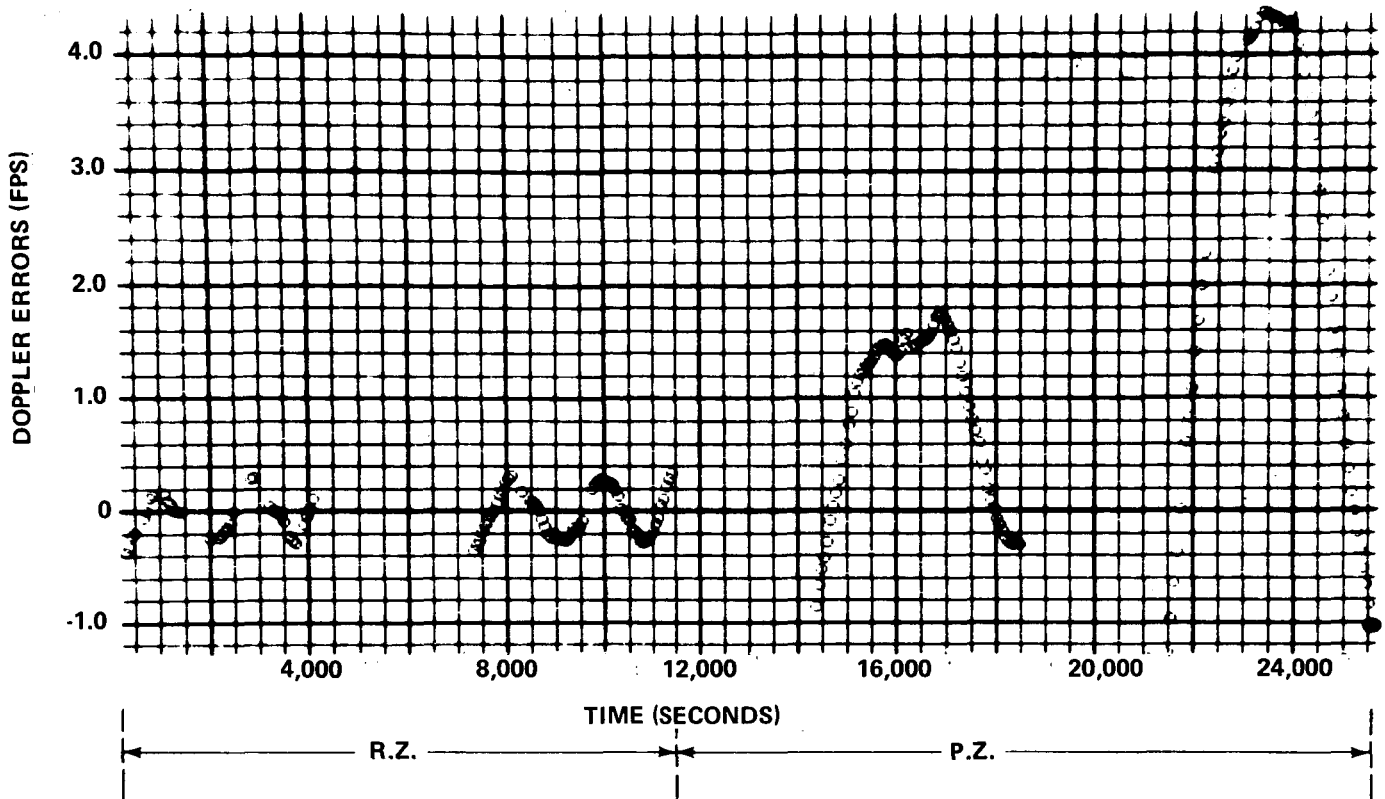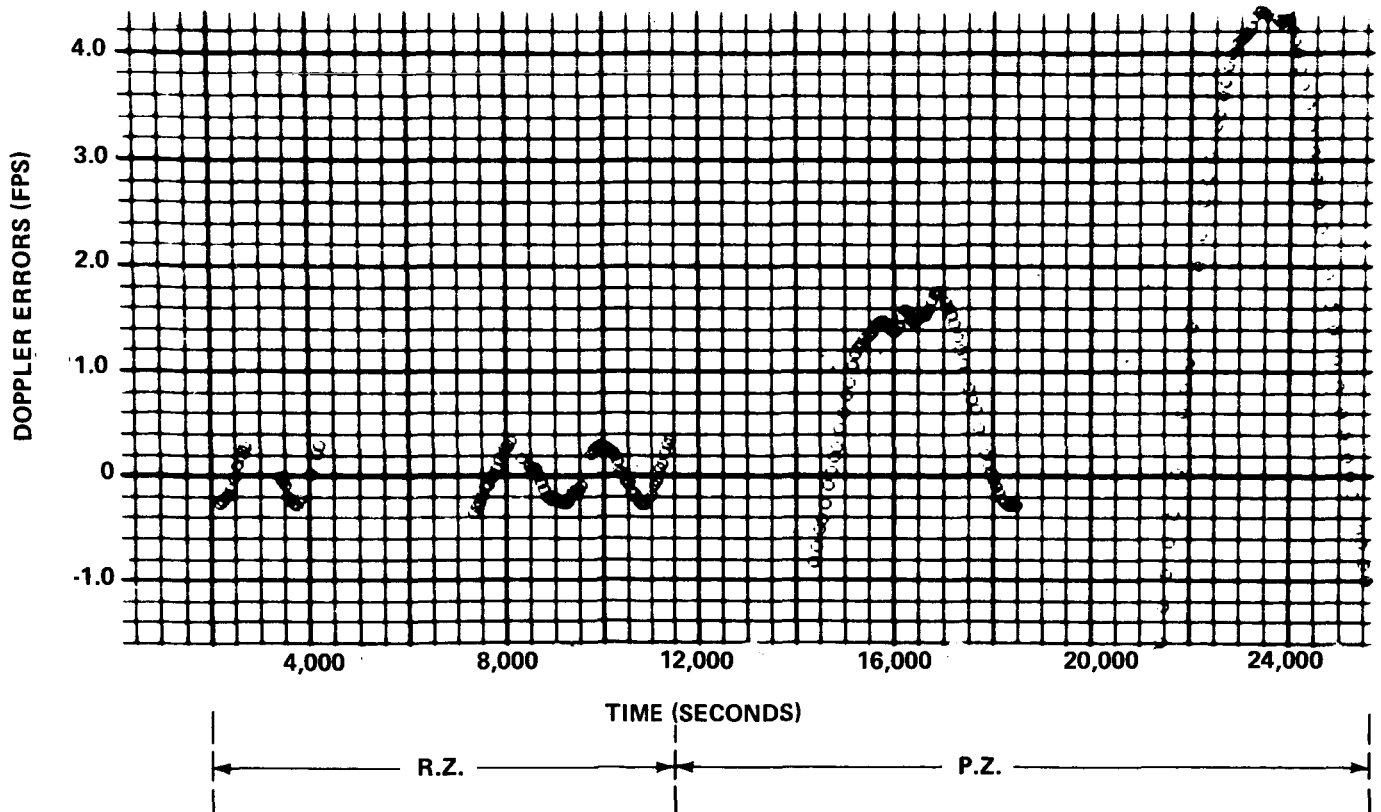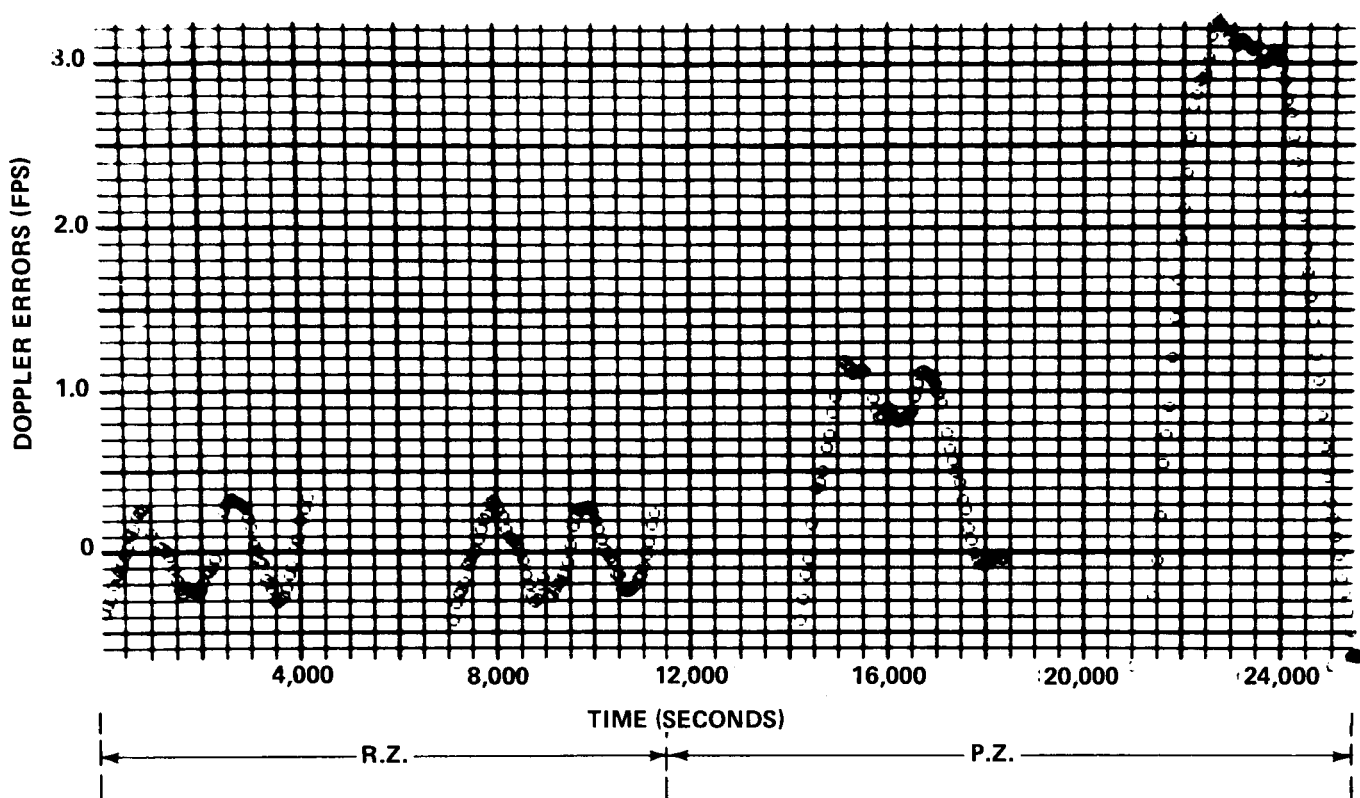PASSES: 3, 4 R.Z./5, 6 P.Z.

FIGURE 1

ASCENSION MSFN

ANTIGUA MSFN
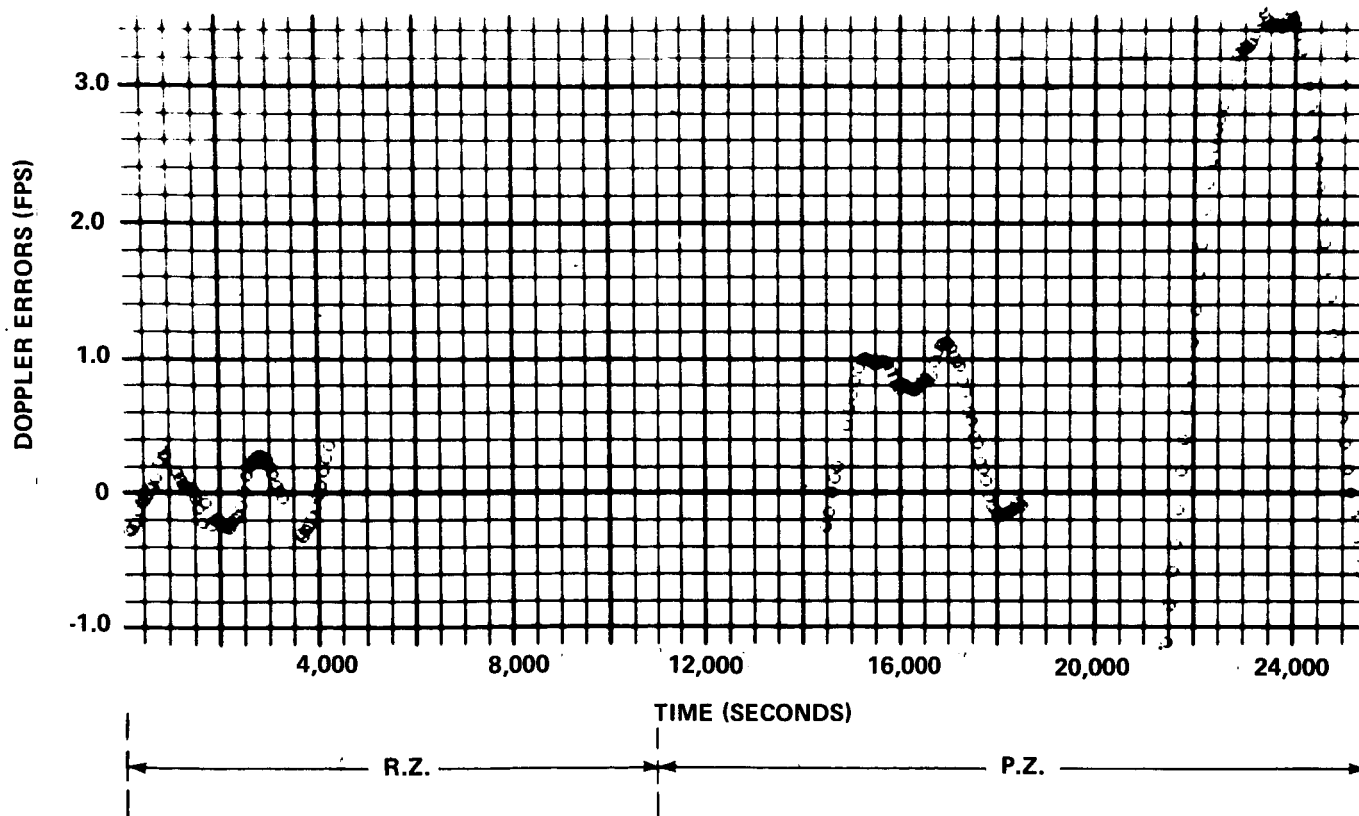
PASSES: 4, 5 R.Z./6, 7 P.Z.
FIGURE 2
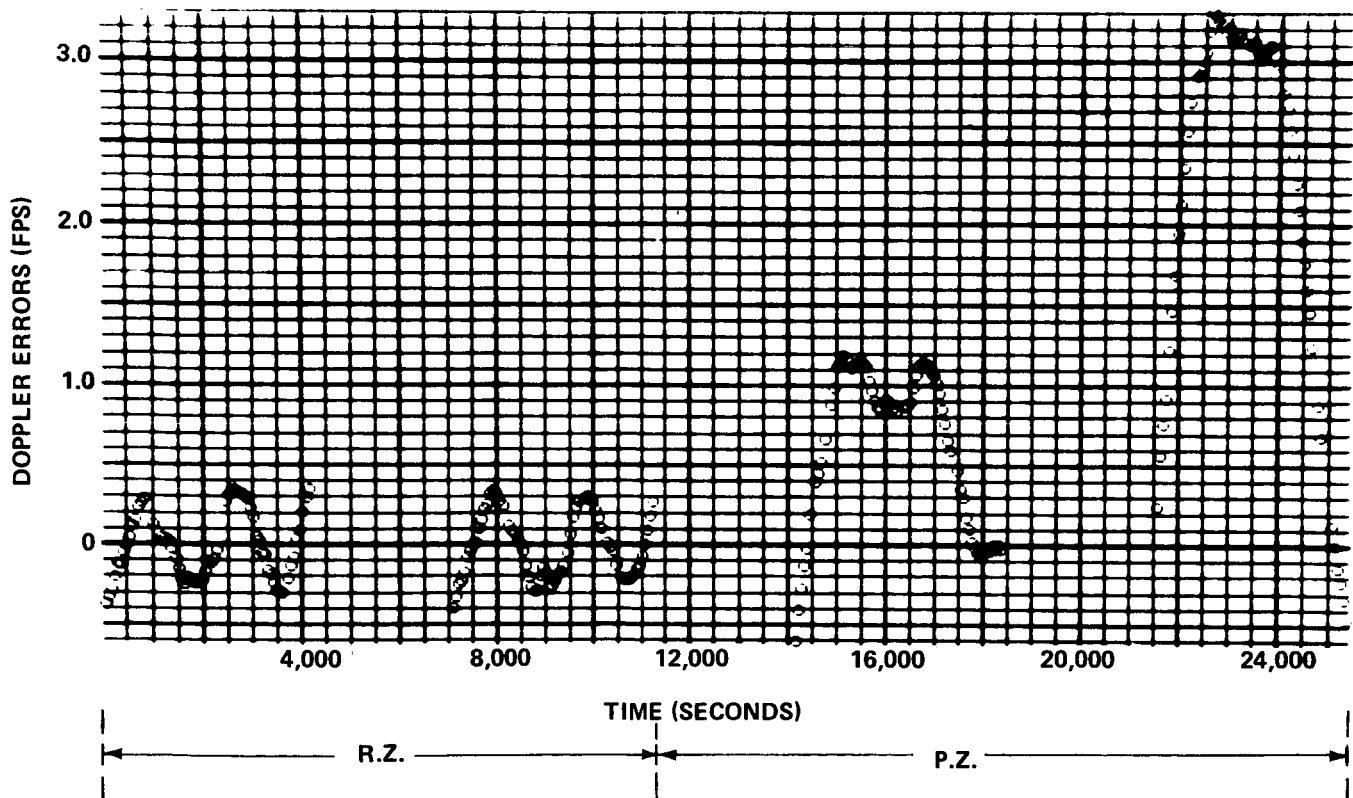
GUAYMAS MSFN

GOLDSTONE MSFN
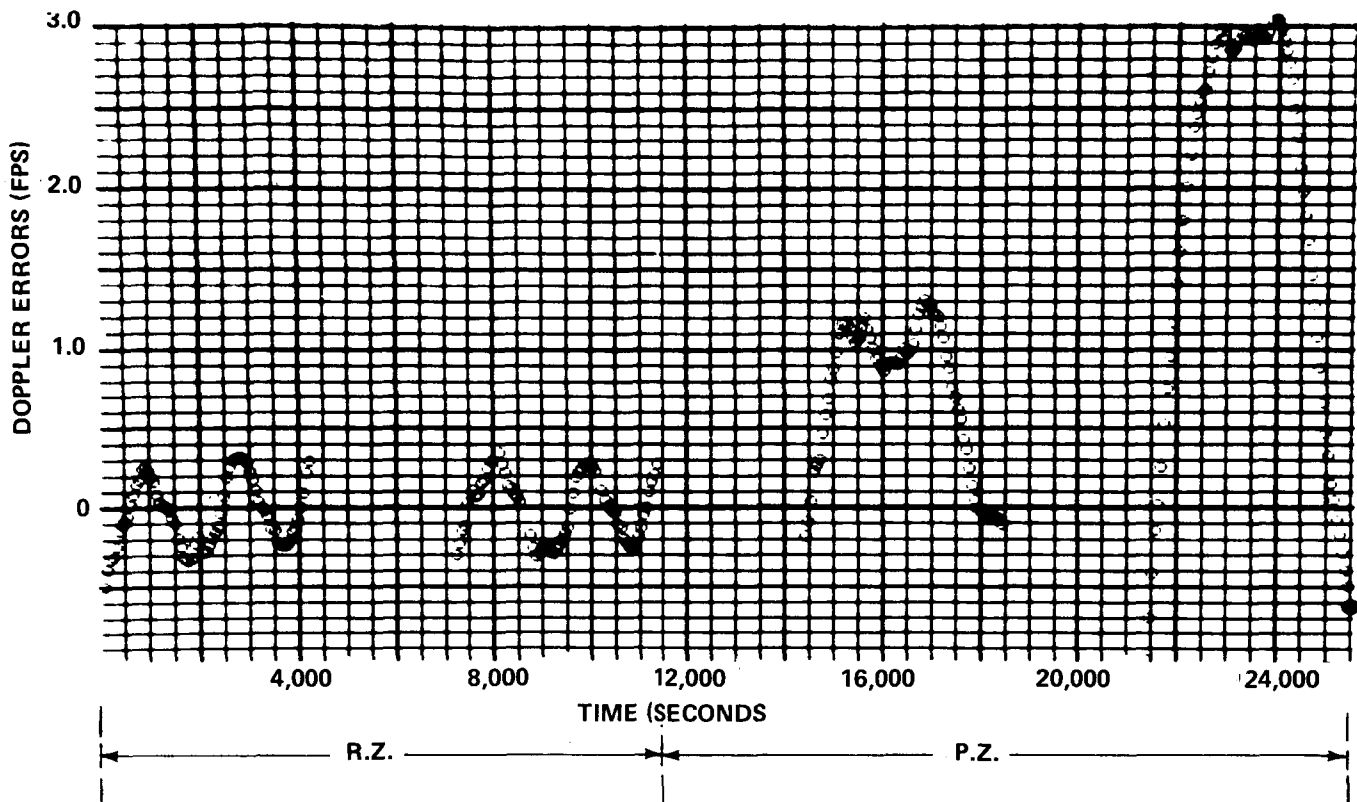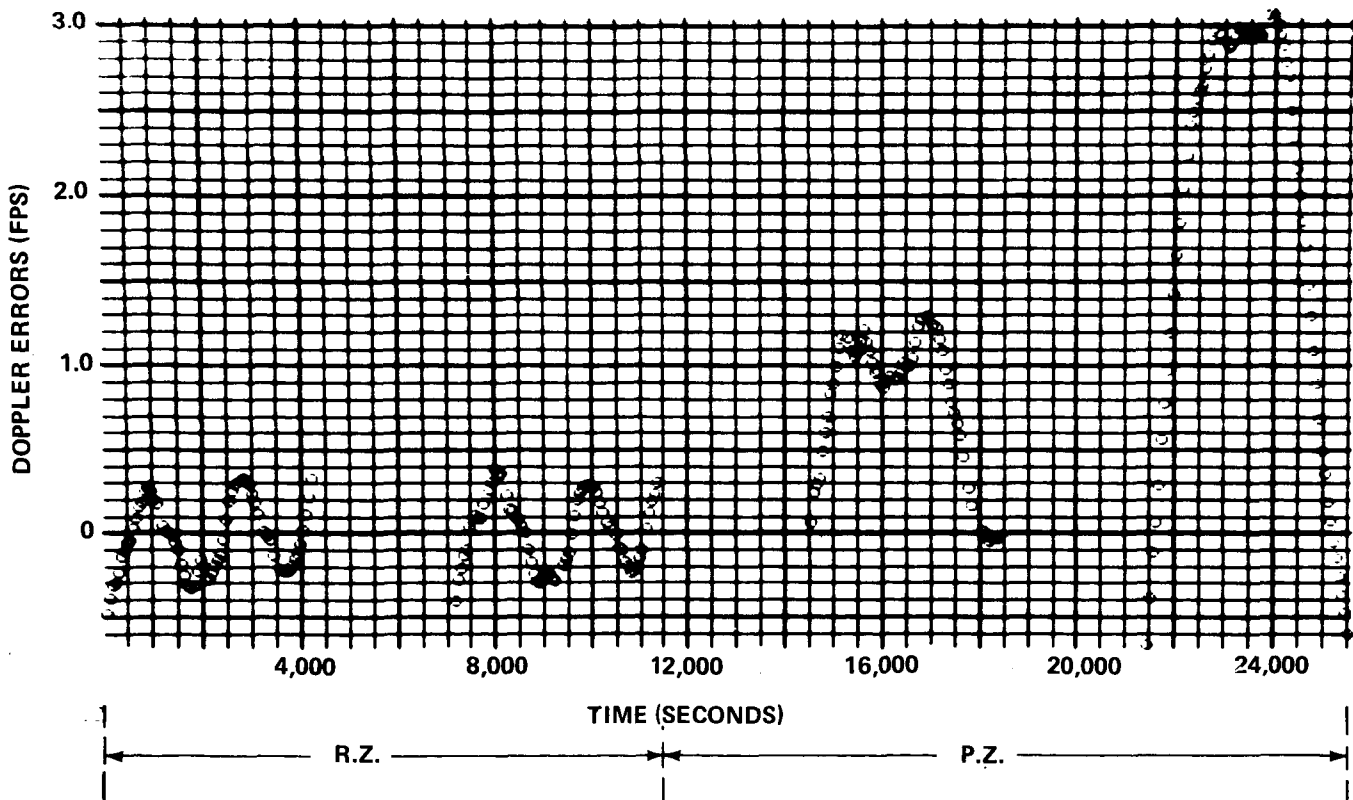
PASSES: 5, 6 R.Z./7, 8 P.Z.
FIGURE 3

# GUAYMAS MSFN



DOPPLER ERRORS (FPS) vs TIME (SECONDS)

4,000    8,000    12,000    16,000    :20,000    24,000

R.Z. ———————— P.Z.

# GOLDSTONE MSFN



DOPPLER ERRORS (FPS) vs TIME (SECONDS)

4,000    8,000    12,000    16,000    20,000    24,000

R.Z. ———————— P.Z.

PASSES:  6, 7 R.Z./8, 9 P.Z.
FIGURE 4

GUAYMAS MSFN

GOLDSTONE MSFN

PASSES: 7, 8 R.Z./9, 10 P.Z.

FIGURE 5

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TITLE- An Analysis of Apollo 8 Tracking Data
Utilizing the Osculating Lunar Elements
Program

TM-69-2014-8

DATE- June 30, 1969

FILING CASE NO(S)- 310

AUTHOR(S)- M. V. Bullock
A. J. Ferrari

FILING SUBJECT(S)- Orbit Determination
(ASSIGNED BY AUTHOR(S)-

### ABSTRACT

An analysis of the eight front side passes (after
the LOI-2 trajectory circularization) of Apollo 8 Doppler
tracking data was made using the Osculating Lunar Elements
Program (O.L.E.P.).  Solutions were obtained on a two pass
regression zone/two pass propagation zone basis.

The two pass solutions obtained, in general, resulted
in propagation errors of about five times those of the regression
zone (on a peak to peak basis).  This growth rate in errors
represents a 25% improvement over standard techniques using
the R2 lunar gravity field (unpublished results).

An examination of some high correlations present
in the solution covariance matrix revealed a lack of sensitivity
in the minimization process.  This lack of sensitivity gives
rise to linear combinations among some estimated parameters
and ultimately leads to a non-unique state solution.  This
loss of uniqueness is responsible for the extrapolation
characteristics attained.

On the basis of the Apollo 8 solutions, it can be
said that the O.L.E.P. concept provides an effective means
for fitting the Doppler observable obtained from lunar orbits.

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